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DETERMINATION OF THE CHARACTER OF THE LATERAL-DIRECTIONAL MOTION OF AN AIRBORNE TOWED VEHICLE

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This report presents the derivation of the lateral-directional equations of motion of an airborne towed vehicle. Herein it is assumed that the tow cable is stable and is attached to the vehicle at other than its center-of-gravity. In addition to customary airborne vehicle equations of motion, the lateral rotation rate of the cable attachment point on the vehicle is described as a function of cable length. Components of cable tension are included in the equations. Determinations of the character of the lateral-directional motion of a HTM-11 and a modified HTM-11 towed vehicles

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) employing the developed equations of motion are discussed.

S U M M A R Y

This report describes the development of the equations of motion of an airborne towed vehicle and the evaluation of its lateral-directional dynamic stability. In the study it is assumed the towing cable is dynamically stable and cable scopes of 150 to 400 feet are deployed.

Determination of the character of the lateral-directional motion of a HTM-11 and a modified HTM-11 towed vehicles employing the developed equations of motion are discussed.

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INTRODUCTION

This report discusses an evaluation of the lateral-directional dynamic stability of an airborne towed vehicle. In so doing, the vehicle's equations of motion are developed. A simultaneous solution of the equations results in a sixth order polynomial characteristic equation. Evaluation and inspection of the roots of the polynomial determines the dynamic stability state of the vehicle.

The equations of motion of the airborne towed vehicle are similar in nature to those equations derived for a fixed wing aircraft in flight. The exception in the case of the towed vehicle is that the equations herein reflect the cable tension on the body due to deployment of various lengths of cable.

The derivation of the equations of motion of a fixed wing aircraft in flight has been published in references (a), (b) and (c). Therefore, a summary of the derivation of those equations and a development of the equations peculiar to the towed vehicle are discussed herein.

The vehicle analyzed is assumed to be a lifting body, which has its tow cable attachment point at a location forward of its center of gravity similar to the sketch contained in Figure 1. It is also assumed that the tow cable is dynamically stable. For a typical mission it is assumed that the vehicle is deployed to cable scopes of 150 to 400 feet.

Two applications of the derived equations of motion to real life, towed body incidents are discussed.

The effort described, herein, was achieved under the authority of reference (d).

EQUATIONS OF MOTION WITH RESPECT TO AXES FIXED IN SPACE

It is assumed in the analysis that: (1) the vehicle's airframe is a rigid body (2) the earth is fixed in space and (3) the mass of the vehicle is constant.

Mainly the equations of motion of the vehicle are based on Newton's second law of motion. This law states that the rate of change of momentum of a body is proportional to the force applied to that body. Also, the rate of change of the moment of momentum of a body is proportional to the torque applied. The summation of the forces and torques applied to the body are referred to a right hand system of Cartesian axes fixed in space as shown in Figure 1.

Mathematically stated these equations of motion are:

$$\Sigma F_{Y_I} = \frac{d(m_W \bar{V})}{dt}$$

$$\Sigma L = \frac{d\bar{h}_{X_I}}{dt}$$

$$\Sigma N = \frac{d\bar{h}_{Z_I}}{dt} \qquad (1)$$

where ΣF_{y_I} and $\bar{V}$ are, respectively, a summation of the external forces applied parallel to the Y axis and a component of the total velocity along the same axis and m_W is the vehicle's mass; where L and N are summations of components of moments about the X_I and Z_I axes, respectively; where h_{X_I} and h_{Z_I} are components of the moment of momentum about the X_I and Z_I axes, respectively.

The components of the moment of momentum are derived by summing the moments of the linear velocity vectors about each axis, multiplying by the differential mass and integrating over the entire mass. The resulting components are:

$$h_{X_I} = PI_{XX} - QI_{XY} - RI_{XZ}$$

$$h_{Z_I} = RI_{ZZ} - PI_{XZ} - QI_{YZ} \qquad (2)$$

where P, Q and R are angular velocities along the X_I, Y_I, Z_I axes, respectively, and I is the moment and product of inertia about the respective subscripted axes.

EQUATIONS OF MOTION WITH RESPECT TO EULERIAN AXES

If the reference axes remain fixed in space and the vehicle rotates, the moments and products of inertia will vary with time and functions of P, Q and R will appear in the equation (1) as derivatives. In order to circumvent the use of these terms an Eulerian axis system is adopted. Therefore, it was assumed that the X, Y, Z axes, as seen in Figure 1 with their origin at the vehicle's center of gravity, are fixed to the vehicle and move with it.

A general expression of equations (1) referred to the Eulerian axes become:

$$\bar{F} = m_W \frac{\partial V_T}{\partial t} + m_W \bar{\omega} \times \bar{V}_T$$

$$\bar{M} = \frac{\partial \bar{h}}{\partial t} + \bar{\omega} \times \bar{h} \qquad (3)$$

where $\bar{\omega} \times \bar{V}_T$ is the centripetal acceleration resulting from the vector cross-product of angular velocity, $\bar{\omega}$, and the linear velocity, V_T; and the vector cross-product $\bar{\omega} \times \bar{h}$ is the absolute rate of change of moment of momemtum.

The expanded form of equations (3) using the scalar components and assuming the XZ plane of the vehicle is a plane of symmetry are:

$$\Sigma F_y = m_W (\dot{V} + RU - PW)$$

$$\Sigma L = \dot{P}I_{XX} - \dot{R}I_{XZ} + QR(I_{ZZ} - I_{YY}) - PQI_{XZ}$$

$$\Sigma N = \dot{R}I_{ZZ} - \dot{P}I_{XZ} + PQ(I_{YY} - I_{XX}) + QRI_{XZ} \quad , \quad (4)$$

LINEARIZING THE EQUATIONS OF MOTION

The complexity of the analysis can be simplified by linearizing the equations of motion. The equations are linearized by assuming that the motion of the vehicle is the result of disturbing it from an equilibrium, steady flight condition and further assuming the perturbations from steady flight are small. The velocities can then be written as:

$$U = U_o + u$$

$$V = V_o + v$$

$$W = W_o + w$$

$$P = P_o + p$$

$$Q = Q_o + q$$

$$R = R_o + r \quad (5)$$

Zero subscripts indicate steady state velocities; lower case letters indicate perturbed quantities. It is also assumed the vehicle is originally in steady flight with the wings level and all components of velocity are zero except U_o and W_o. Then

$$P_o = Q_o = R_o = V_o = 0 \quad (6)$$

Expanding equations (4) by incorporating equations (5) and (6) and assuming that the perturbations are small enough that the products and squares of the velocity changes are negligible, the equations of motion reduce to:

$$\Sigma F_Y = m(\dot{v} + U_o r - W_o p)$$

$$\Sigma L = \dot{p}I_{XX} - \dot{r}I_{XZ}$$

$$\Sigma N = \dot{r}I_{ZZ} - \dot{p}I_{XZ} \quad (7)$$

DEVELOPMENT OF GRAVITY FORCE COMPONENT

The gravity force contributes components to the summation of external forces applied to the vehicle. In this report the external forces are limited to contributions along the vehicle's Y axis. The gravity components act along steady flight axes which it is assumed has an initial pitch angle displacement, θ_o, with respect to the gravity vector. Resolving the gravity force along the steady flight axes produces:

$$W_{o_X} = - m_W g \sin \theta_o$$

$$W_{o_Y} = o$$

$$W_{o_Z} = m_W g \cos \theta_o \tag{8}$$

where g is the gravity acceleration constant. It is further assumed that the axes are disturbed and displaced from the steady axes by Eulerian angles ψ, θ ϕ, in that order of rotation.

Assuming that the disturbed Eulerian angles and θ_o are small, and the cosine of the angles equal one and the sine of the angles equal the angle, the products of these angles are approximately zero and can be neglected. Therefore, the gravity force component acting along the disturbed Eulerian Y axis is:

$$F_{Y_g} = \phi m_W g \tag{9}$$

DEVELOPMENT OF THE AERODYNAMIC FORCE AND MOMENTS

The aerodynamic forces and moments applied to the airborne towed vehicle are due to the resistance of the surrounding atmosphere to the vehicle's motion.

The expression of the aerodynamic force along the vehicle's Y axis can be written as:

$$Y_F = C_Y \; 1/2 \; \rho \; V_T^2 S_W \tag{10}$$

where C_Y is a dimensionless coefficient, ρ is the atmospheric density, V_T is the velocity of the vehicle, and S_W is the vehicle's wing area.

The rolling and yawing moment about the vehicle's X and Z axis can be expressed respectively as:

$$L = C_{\ell} \; 1/2 \, \rho \, V_T^2 \, S_W b$$

and

$$N = C_n \; 1/2 \, \rho \, V_T^2 \, S_W b \tag{11}$$

where C_{ℓ} and C_n, are dimensionless coefficients and b is the vehicle's wing span.

The forces and moments acting on the disturbed vehicle can be written as a function of the parameters with which each is known to vary by expanding the terms in a Taylor series.

Therefore, the force Y_F and moments L and N can be written as:

$$Y_F = Y_{F_O} + \frac{\partial Y_F}{\partial r} r + \frac{\partial Y_F}{\partial \dot{r}} \dot{r} + \frac{\partial Y_F}{\partial v} v + \frac{\partial Y_F}{\partial \dot{v}} \dot{v} + \frac{\partial Y_F}{\partial p} p + \frac{\partial Y_F}{\partial \dot{p}} \dot{p}$$

$$L = L_O + \frac{\partial L}{\partial r} r + \frac{\partial L}{\partial \dot{r}} \dot{r} + \frac{\partial L}{\partial v} v + \frac{\partial L}{\partial \dot{v}} \dot{v} + \frac{\partial L}{\partial p} p + \frac{\partial L}{\partial \dot{p}} \dot{p}$$

$$N = N_O + \frac{\partial N}{\partial r} r + \frac{\partial N}{\partial \dot{r}} \dot{r} + \frac{\partial N}{\partial v} v + \frac{\partial N}{\partial \dot{v}} \dot{v} + \frac{\partial N}{\partial p} p + \frac{\partial N}{\partial \dot{p}} \dot{p} \tag{12}$$

The zero subscript indicates the portions of the force and moments which are acting during steady flight. The partial derivative terms express the forces and moments created by the disturbed parameters. In this case due to steady flight assumptions $Y_{F_O} = L_O = N_O = 0$. It is assumed that the air flow about the vehicle is quasi-steady, therefore all derivatives with respect to the rate of change of velocities are omitted.

DEVELOPMENT OF THE CABLE TENSION FORCE COMPONENT AND MOMENTS

It is assumed in this analysis that the tow cable discussed herein is dynamically stable.

The orientation of the cable tension vector relative to the towed vehicle's axis system is presented in Figure 2.

Cable tension contributes components to the summation of applied external forces and moments and is assumed to act along the longitudinal axis of the cable.

The steady state components of the tension which have an initial pitch angle displacement θ_o and a lateral angle displacement of σ are determined by resolution of the tension vector along the steady state axes X_o, Y_o and Z_o.

The steady state components of the cable tension are:

$$T_{o_X} = T \cos \theta_c - \theta_o T \sin \theta_c$$

$$T_{o_Y} = -\sigma T \cos \theta_c$$

$$T_{o_Z} = \theta_o T \cos \theta_c + T \sin \theta_c \quad (13)$$

where angles θ_o and σ are assumed to be small and T is the tension of the cable exerted on the drogue during trimmed steady flight. The component of cable tension acting along the disturbed Y Eulerian axis is determined by performing an Euler angle transformation and employing the small angle assumptions. Therefore

$$F_{Y_T} = -\Psi T_{o_X} + T_{o_Y} + \phi T_{o_Z} \quad (14)$$

Expanding F_{Y_T} by substituting equations (13) and simplifying gives:

$$F_{Y_T} = -\psi T \cos \theta_c - \sigma T \cos \theta_c + \phi T \sin \theta_c \quad (15)$$

The cable attachment point (tow point) on the towed vehicle is remote from the vehicle's center of gravity as shown in Figure 1, therefore moments about the X and Z disturbed Eulerian axes are generated by the cable tension component, F_{Y_T}, in equation (15). The moments are:

$$L_T = (-\psi T \cos \theta_c - \sigma T \cos \theta_c + \phi T \sin \theta_c) z_{TP}$$

and

$$N_T = (-\psi T \cos \theta_c - \sigma T \cos \theta_c + \phi T \sin \theta_c) x_{TP} \quad (16)$$

SUMMATION OF EXTERNAL FORCES AND MOMENT

The external forces applied to the towed vehicle are: gravity force, aerodynamic forces and cable force.

Substituting the forces acting along the disturbed Eulerian Y axis from equations (9), (12) and (14) into the force equation of equations (7) and rearranging results in the following:

$$\dot{v} + U_o r - W_o p - \phi g = Y_r r + Y_v v + Y_p p - \frac{\psi T \cos\theta_c}{m_W} - \frac{\sigma T \cos\theta_c}{m_W} + \frac{\phi T \sin\theta_c}{m_W}$$

where $\frac{1}{m_W}\frac{\partial Y_F}{\partial r}$, $\frac{1}{m_W}\frac{\partial Y_F}{\partial v}$ and $\frac{1}{m_W}\frac{\partial Y_F}{\partial p}$ are replaced

by Y_r, Y_v and Y_p. These terms are further defined in Table I. (17)

Substituting the moments about the disturbed Eulerian X and Z axis from equations (12) and (16) into the moment equations of equations (7) and rearranging the terms, results in the following equations:

$$\dot{p} - \dot{r}\frac{I_{XZ}}{I_{XX}} = L_v v + L_p p + L_r r - \frac{z_{TP}\psi\, T \cos\theta_c}{I_{XX}} - \frac{z_{TP}\sigma\, T \cos\theta_c}{I_{XX}} + \frac{z_{TP}\phi\, T \sin\theta_c}{I_{XX}}$$

$$\dot{r} - \dot{p}\frac{I_{XZ}}{I_{ZZ}} = N_v v + N_p p + N_r r - \frac{x_{TP}\psi\, T \cos\theta_c}{I_{ZZ}} - \frac{x_{TP}\sigma\, T \cos\theta_c}{I_{ZZ}} + \frac{x_{TP}\phi \quad \sin\theta_c}{I_{ZZ}} \qquad (18)$$

where $\frac{1}{I_{XX}}\frac{\partial L}{\partial v}$ and $\frac{1}{I_{ZZ}}\frac{\partial N}{\partial v}$ are replaced by L_v and N_v.

Similar substitutions are made for functions of p and r. L_v, L_p, L_r, N_v, N_p and N_r are defined in Table I.

RELATIONSHIP BETWEEN EULER ANGLES AND ANGULAR VELOCITIES

As mentioned previously, the Eulerian axes are displaced from the steady flight axes by the Eulerian angles ψ, θ and ϕ. The rates of changes of the Eulerian angles can be represented as vectors along the axes about which the rotations take place.

The following lateral-directional equations relating these terms are:

$$\dot{\phi} = P + Q \tan\theta \sin\phi + R \tan\theta \cos\phi$$

$$\dot{\psi} = R\left(\frac{\cos\phi}{\cos\theta}\right) + Q\left(\frac{\sin\phi}{\cos\theta}\right) \qquad (19)$$

Restricting the analysis to the lateral-directional mode and assuming that the perturbations evolve from a straight and level, steady flight condition, $P_o = Q_o = R_o = \psi_o = \phi_o = q = 0$ and $\theta_o = \alpha_o$ and are of small magnitude.

Equations (19) reduce to

$$\dot{\phi} = p + r \tan \theta_o \qquad (20)$$

$$\dot{\psi} = r$$

DEVELOPMENT OF TOW CABLE ANGULAR ROTATION RATE

The absolute linear velocity of the towpoint with respect to the towed vehicle's center of gravity is expressed as:

$$\bar{V}_{TP} = \bar{V}_{cg} + \bar{V}_{TP/cg} \qquad (21)$$

where $\bar{V}_{cg}$ is the linear velocity vector of the center of gravity and $V_{TP/cg}$ is the linear velocity vector of the tow point with respect to the center of gravity. Equation (21) can also be expressed as:

$$\bar{V}_{TP} = \bar{V}_{cg} + \bar{\omega} \times \bar{r}_c \qquad (22)$$

Recalling that the components of the vehicle's angular velocity vector $\bar{\omega}$ are $\bar{P}$, $\bar{Q}$ and $\bar{R}$ and the components of $\bar{r}_c$ (distance between cg and towpoint) are x_{TP} y_{TP} and z_{TP}; the cross product of equation (22) can be expanded by the following determinant:

$$\bar{\omega} \times \bar{r}_c = \begin{vmatrix} i & j & k \\ P & Q & R \\ x_{TP} & y_{TP} & -z_{TP} \end{vmatrix} \qquad (23)$$

Expanding the determinant

$$\bar{\omega} \times \bar{r} = i\,(-Qz_{TP} - Ry_{TP}) + j\,(Pz_{TP} + Rx_{TP}) + k\,(Py_{TP} - Qx_{TP}) \qquad (24)$$

The component of towpoint velocity along the vehicle's Y axis is:

$$\dot{Y}_{TP} = \dot{Y}_{cg} + pz_{TP} + rx_{TP} \qquad (25)$$

where as before $P_o = R_o = o$. The inertial velocity component $\dot{Y}_{cg}$ can be expressed in terms of disturbed axis velocity components by using the following Euler angle transformation:

$$\dot{Y}_{cg} = U \cos \theta \sin \psi + V (\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi)$$
$$+ W (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \tag{26}$$

Recalling that ϕ, ψ, θ are small, expanding the velocity components, as during the linearization, assuming V_o is zero and neglecting resulting small terms, $\dot{Y}_{cg}$ can be written as:

$$\dot{Y}_{cg} = U_o \psi + v - W_o \phi \tag{27}$$

Substituting $\dot{Y}_{cg}$ into equation (25)

$$\dot{Y}_{TP} = U_o \psi + v - W_o \phi + p z_{TP} + r x_{TP} \tag{28}$$

The vehicle is attached to the towing aircraft as shown in Figure 2. The tow cable of length, ℓ, generates an angle, σ, in the XY inertial plane when the vehicle rotates about the cable attachment point on the aircraft. The inertial linear velocity, $\dot{Y}_{TP}$, therefore, also can be expressed as:

$$\dot{Y}_{TP} = \dot{\sigma}\, \ell \cos \theta_c \tag{29}$$

Equating the equations (28) and (29) and rearranging, the expression for $\dot{\sigma}$ is:

$$\dot{\sigma} = \frac{U_o \psi + v - W_o \phi + p z_{TP} + r x_{TP}}{\ell \cos \theta_c} \tag{30}$$

AXIS SYSTEM

A body axis system is chosen where the X axis is oriented along the longitudinal principal axis of the vehicle. The product of inertia, I_{XZ}, used in equations (31) therefore is equal to zero.

LATERAL-DIRECTIONAL EQUATIONS OF MOTION

A recapitulation of the rearranged equations of motion of the disturbed towed vehicle taken from equations (17), (18), (20) and (30) is:

$$\dot{v} - Y_v v - (Y_p + W_o) p + (U_o - Y_r) r - \left(g + \frac{T \sin \theta_c}{m_W}\right) \phi$$
$$+ \frac{T \cos \theta_c}{m_W} \psi + \frac{T \cos \theta_c}{m_W} \sigma = 0$$

$$- L_v v + \dot{p} - L_p p - L_r r - \frac{z_{TP} T \sin\theta_c}{I_{XX}} \phi + \frac{z_{TP} T \cos\theta_c}{I_{XX}} \psi +$$

$$\frac{z_{TP} T \cos\theta_c}{I_{XX}} \sigma = 0$$

$$- N_v v - N_p p + \dot{r} - N_r r - \frac{x_{TP} T \sin\theta_c}{I_{ZZ}} \phi + \frac{x_{TP} T \cos\theta_c}{I_{ZZ}} \psi +$$

$$\frac{x_{TP} T \cos\theta_c}{I_{ZZ}} \sigma = 0 \qquad (31)$$

$$p + r \tan_{\phi_o} - \dot{\phi} = 0$$

$$r - \dot{\psi} = 0$$

$$\frac{- v - z_{TP} p - x_{TP} r + W_o \phi - U_o \psi}{\ell \cos\theta_c} + \dot{\sigma} = 0$$

CHARACTER OF MOTION OF DISTURBED VEHICLE

A simultaneous solution of the equations of motion results in a sixth order polynomial characteristic equation. The character of the lateral-directional motion of the towed vehicle deployed at various cable lengths is revealed by the roots of the polynomial. If the roots are real numbers, the motion is aperiodic. This motion is convergent if the roots are negative, divergent if positive. When the roots form a complex pair, the motion of the vehicle is oscillatory. Damped oscillatory motion prevails if the real part of the root is negative, undamped motion if the real part is positive.

VALIDITY OF METHOD

In order to establish credence in the ability of the equations of motion and resulting characteristic equation to determine the lateral-directional dynamic stability of a given vehicle, two applications of the method are discussed.

During a typical deployment of an HTM-11 airborne towed vehicle, a drogue used to carry electronic equipment, severe lateral, undamped oscillations occurred. Oscillations of the vehicle were severe enough to separate the towing cable. The incident occurred at approximately 210 KIAS at an altitude of 10,000 feet. Cable deployed at the time of the incident was approximately three to seven feet in length.

The pertinent mass properties and physical characteristics of the HTM-11 are noted in Table II. Static aerodynamic coefficients which are listed in Table III are based on wind tunnel data determined at the Naval Ship Research and Development Facility. Dynamic aerodynamic coefficients were derived at the Naval Air Development Center. The perturbed motion in the application was assumed to evolve from the derived trim conditions of Table IV.

A characteristic equation was ultimately determined after applying the values noted in the tables to equations (31). Three sets of complex roots were evaluated for cable lengths between 1 and 300 feet. A root locus of the results is presented in Figure 3. The portion of the locus appearing to the right of the vertical imaginary axis indicates that unstable vehicle flight existed. The locus also revealed that an unstable natural frequency of 7.5 radians per second was evident at a cable scope of 5 feet. In comparison, an unstable natural frequency of 7.8 radians per second at a similar cable length was measured from a video tape recording of the unstable incident of the HTM-11.

Further investigation employing the above method revealed that by extending the vertical end plates of the wing of the HTM-11 downward, the lateral dynamic stability of the vehicle would be stabilized. Aerodynamic characteristics of the modified HTM-11, determined in the wind tunnel at the Naval Ship Research and Development Facility and derived at the Naval Air Development Center are listed in Table III. Derived equilibrium trim conditions resulting from the modification are shown in Table IV. Using the above data in the equations of motion and ultimately solving for the characteristic roots at various cable lengths, the root locus plot presented in Figure 4 was determined. Three sets of stable complex roots representing the characteristic equation of the modified HTM-11 can be seen.

Subsequent flight tests of the modified HTM-11 supported the findings of the study.

CONCLUSION

The equations of motion discussed in this report can be used to determine the lateral-directional stability of a lifting vehicle towed behind an aircraft. It is assumed that the tow cable is deployed to lengths up to 400 feet and is dynamically stable. Employing the equations, the stability of the towed vehicle can be evaluated at various cable lengths.

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LIST OF ABBREVIATIONS AND SYMBOLS

b	wing span
C_ℓ	rolling moment coefficient
C_n	yawing moment coefficient
C_y	side force coefficient
C_{ℓ_r}	rolling moment coefficient change due to variation in yawing velocity
C_{ℓ_p}	rolling moment coefficient change due to variation in rolling velocity
C_{ℓ_β}	rolling moment coefficient change due to variation in sideslip
C_{n_r}	yawing moment coefficient change due to variation in yawing velocity
C_{n_p}	yawing moment coefficient change due to variation in rolling velocity
C_{n_β}	yawing moment coefficient change due to variation in sideslip
C_{y_r}	side force coefficient change due to variation in yawing velocity
C_{y_p}	side force coefficient change due to variation in rolling velocity
C_{y_β}	side force coefficient change due to variation in sideslip
$\bar{c}$	mean aerodynamic chord
cg	center of gravity
cm	centimeter
$\frac{d}{dt}$	derivative with respect to time
deg	degree

F	force
ft	feet
g	gravitational constant, 32.2 ft/sec^2 (980 cm/sec^2)
h	moment of momentum
I_{XX}	mass moment of inertia with respect to vehicle's x axis
I_{XY}	product of mass moment of inertia with respect to vehicle's X and Y axes
I_{XZ}	product of mass moment of inertia with respect to vehicle's X and Z axes
I_{ZZ}	mass moment of inertia with respect to vehicle's Z axis
i	unit vector along X axis of vehicle
j	unit vector along Y axis of vehicle
$j\omega$	imaginary segment of complex root
k	unit vector along Z axis of vehicle
kg	kilograms
ℓ	tow cable length
lbs	pounds
m_W	mass of vehicle
m	metre
P	rolling velocity
p	perturbed rolling velocity
Q	pitching velocity
q	perturbed pitching velocity
R	yawing velocity
rad	radian
r	perturbed yawing velocity

S_W	wing area
sec	seconds
U	linear velocity component along vehicle's X axis $_s$
T	cable tension
u	perturbed linear velocity along vehicle's X axis
V	linear velocity component along vehicle's Y axis
V_T	total linear velocity
v	perturbed linear velocity along vehicle's Y axis
W	linear velocity component along vehicle's Z axis
w	perturbed linear velocity along vehicle's Z axis
x_{TP}	distance between vehicle's tow point and cg along vehicle's X axis
y_{TP}	distance between vehicle's tow point and cg along vehicle's Y axis
z_{TP}	distance between vehicle's tow point and cg along vehicle's Z axis
α	trim angle of attack
β	sideslip
θ	Euler angle displacement in pitch
θ_c	cable angle referenced to equilibrium relative wind
θ_o	equivalent to angle of attack during level flight
Σ	summation of terms
σ	cable rotation projected in $X_I Y_I$ plane
σ_r	real segment of complex root
ϕ	Euler angle displacement in roll
Ψ	Euler angle displacement in yaw
ω	angular velocity

$\cdot$	derivative of variable with respect to time
$-$	vector quantity
$\frac{\partial}{\partial t}$	partial derivative of variable with respect to time

Subscripts

F	force
I	respect to inertial axis
T	respect to trimmed flight
Y	respect to Y axis
0	initial conditions

U N I T S

United States (U.S.) customary units were used in this study, but in the reporting, both metric and U.S. customary units appear. Metric units are enclosed by parenthesis.

TABLE I

LATERAL-DIRECTIONAL STABILITY DERIVATIVES

$$L_r = \frac{\rho V_T S_W b^2}{4I_{XX}} C_{\ell_r} \qquad N_v = \frac{\rho V_T S_W b}{2I_{ZZ}} C_{n_\beta}$$

$$L_v = \frac{\rho V_T S_W b}{2I_{XX}} C_{\ell_\beta} \qquad N_p = \frac{\rho V_T S_W b^2}{4I_{ZZ}} C_{n_p}$$

$$L_p = \frac{\rho V_T S_W b^2}{4I_{XX}} C_{\ell_p} \qquad Y_r = \frac{\rho V_T S_W b}{4m_W} C_{Y_r}$$

$$N_r = \frac{\rho V_T S_W b^2}{4I_{ZZ}} C_{n_r} \qquad Y_v = \frac{\rho V_T S_W}{2m_W} C_{Y_\beta}$$

$$Y_p = \frac{\rho V_T S_W b}{4m_W} C_{Y_p}$$

TABLE II

PHYSICAL AND MASS PROPERTIES

b	4.0 ft (1.219m)	S_W	5.50 ft^2 (0.51m^2)
$\bar{c}$	1.375 ft (0.419m)	Weight	225. (lbs) (102.kg)
I_{XX}	5.46 slugs-ft^2 (7.40 kg-m^2)	x_{TP}	2.916 ft (0.889m)
I_{ZZ}	49.26 slugs-ft^2 (66.80 kg-m^2)	z_{TP}	0.812 ft (0.248m)

TABLE III

LATERAL-DIRECTIONAL AERODYNAMIC COEFFICIENTS

HTM-11			HTM-11 (MODIFIED)		
$C_{\ell_\beta}\left(\frac{1}{rad}\right)$	$C_{n_\beta}\left(\frac{1}{rad}\right)$	$C_{Y_\beta}\left(\frac{1}{rad}\right)$	$C_{\ell_\beta}\left(\frac{1}{rad}\right)$	$C_{n_\beta}\left(\frac{1}{rad}\right)$	$C_{Y_\beta}\left(\frac{1}{rad}\right)$
-0.155	0.802	-2.349	-0.0287	1.347	-2.607
$C_{\ell_p}\left(\frac{1}{rad}\right)$	$C_{\ell_r}\left(\frac{1}{rad}\right)$	$C_{n_r}\left(\frac{1}{rad}\right)$	$C_{\ell_p}\left(\frac{1}{rad}\right)$	$C_{\ell_r}\left(\frac{1}{rad}\right)$	$C_{n_r}\left(\frac{1}{rad}\right)$
-0.272	0.029	-1.016	-0.280	0.070	-1.399

TABLE IV

TRIM CONDITIONS

α (deg.)	θ_c (deg.)	T (lbs.)	α (deg.)	θ_c (deg.)	T (lbs.)
-1.78°	-41.11	246.90	-2.20	-37.80	250.83

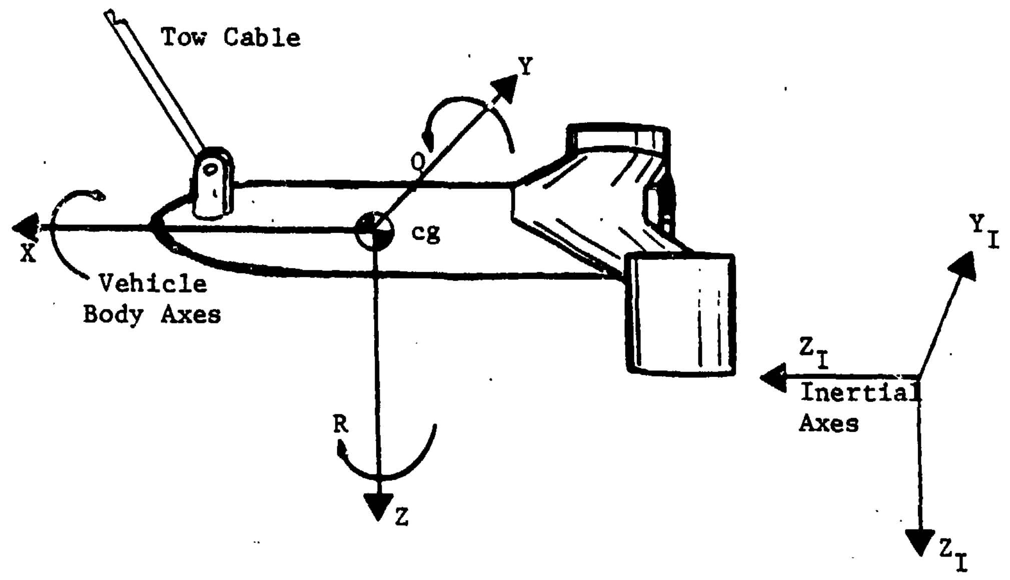

FIGURE 1 AXES SYSTEMS

Cable Angle Sign Convention:
Vehicle below towing aircraft θ_c (-)
Vehicle to right of towing aircraft σ (+)
(looking forward)

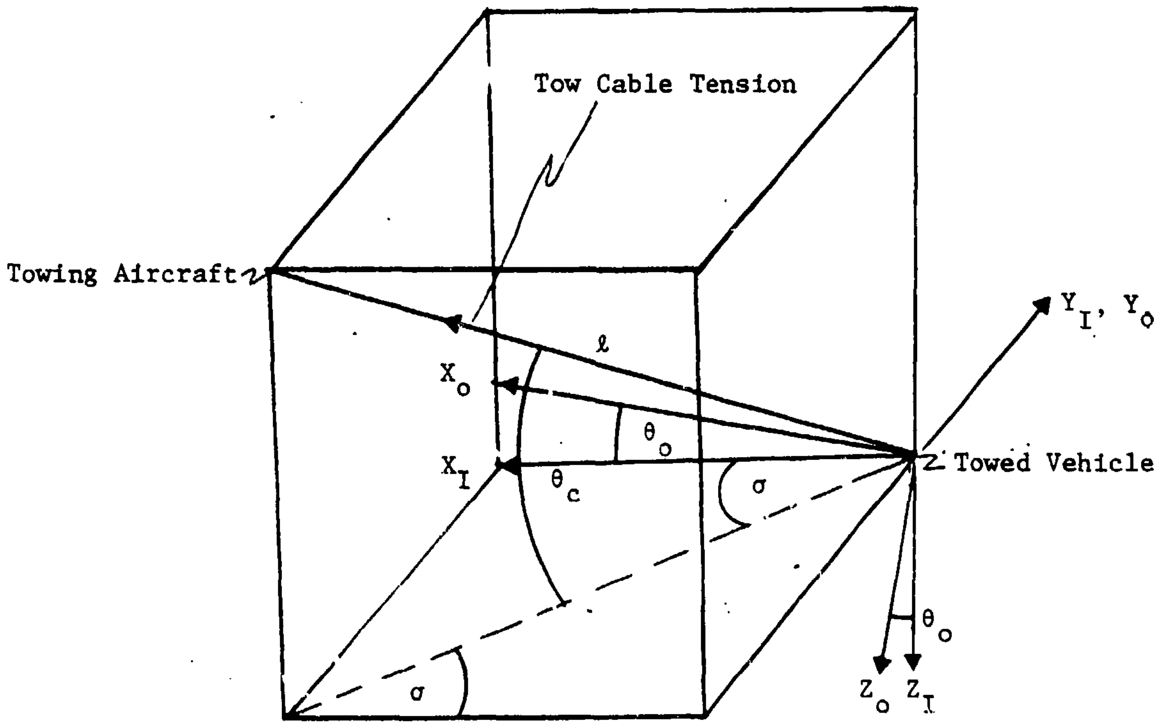

FIGURE 2 ORIENTATION OF CABLE TENSION

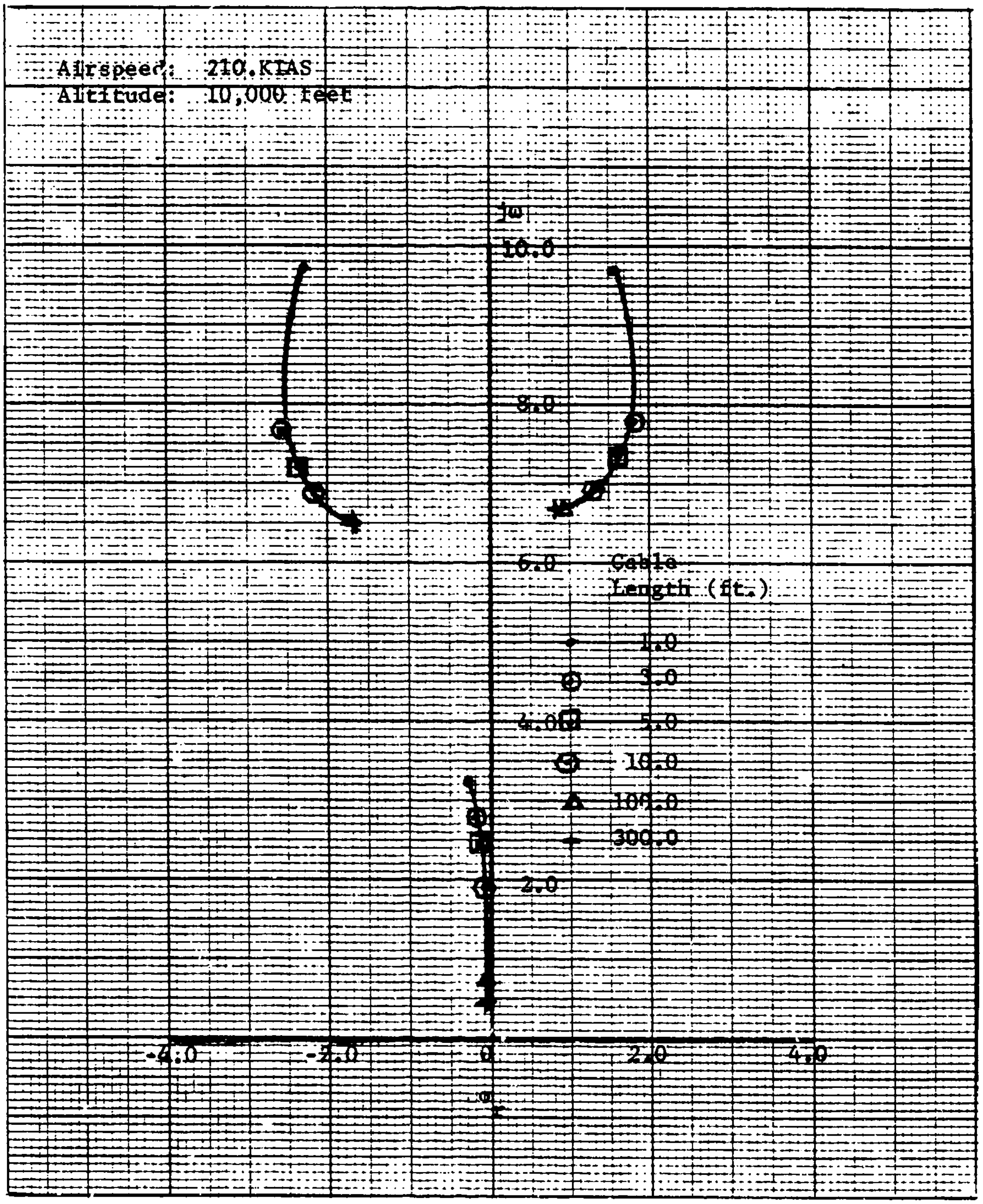

FIGURE 3 ROOT LOCUS - HTM-11 CHARACTERISTIC EQUATION

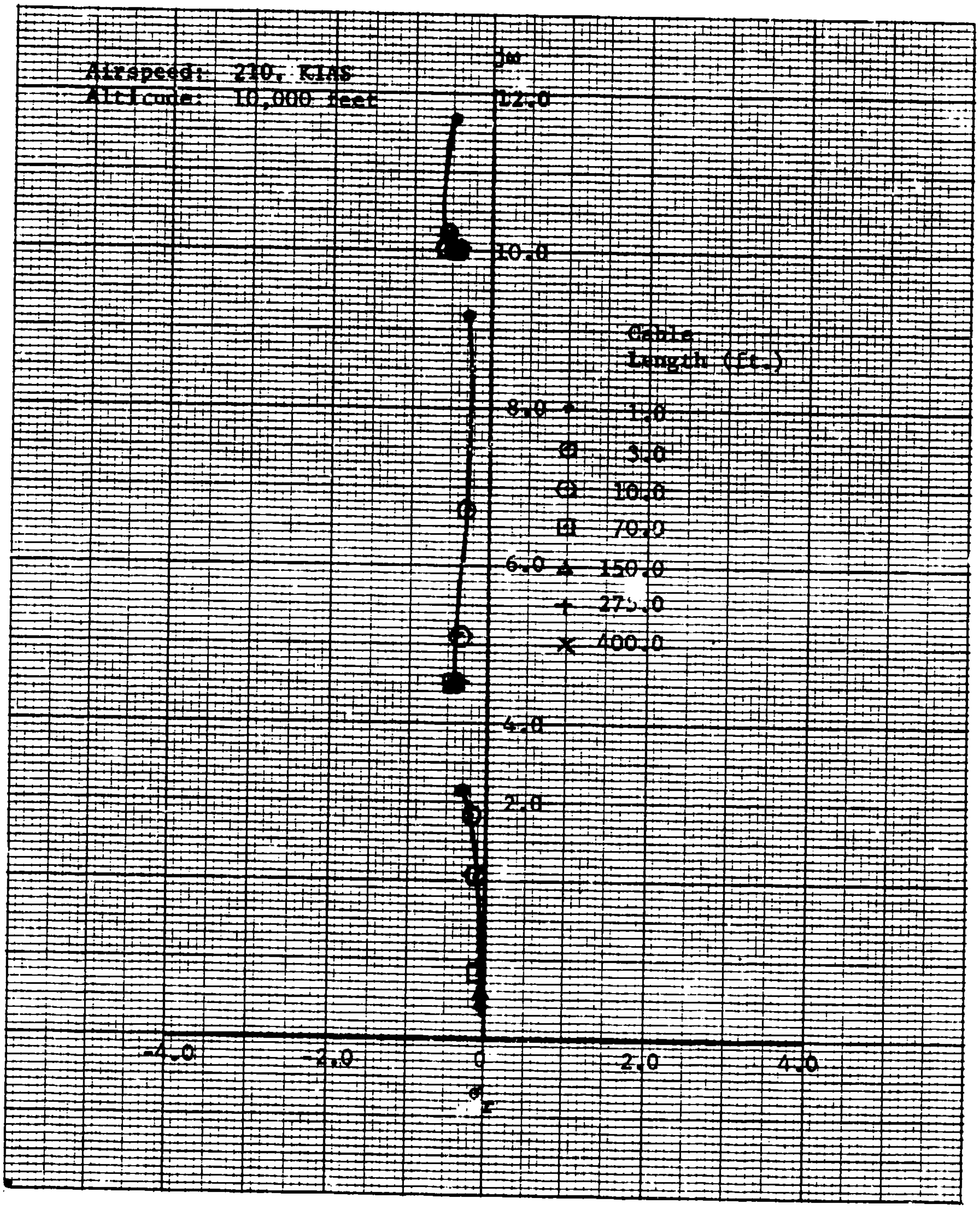

FIGURE 4 ROOT LOCUS - MODIFIED HTM-11 CHARACTERISTIC EQUATION

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